

Division \rightarrow Math.

Math \rightarrow Sum

शुद्ध
वर्त
:
:
+
X
:
:
:

68 172 11
56 42
79 54 53
5372 11#

69 68 12
87 72 58
6003 42 60
12#

$$\frac{7}{2} = 3 \frac{1}{2}$$

$$2x + 3 = 7 \Rightarrow x = 2$$

1	2	3	4	5	6	7	8	9	10
2	4	8	16	32	64	128	256	512	1024
3	9	27	81	243					
4	16	64	256						
5	25	125	625						
6	36	216							
7	49	343							
8	64	512							
9	81	729							
10	100	1000							
11	121	1331							
12	144								
13	169								
14	196								
15	225								
16	256								
17	289								
18	324	324							
19	361								
20	400								

$$200 \times 30 \times 400 = 2400000$$

$$20 \times 100 \times 30 = 60000$$

$$\frac{1276}{10} = 127.6$$

$$\frac{1276}{100} = 12.76$$

$$\frac{1276}{1000} = 1.276$$

$$\frac{1276}{100000} = 0.01276$$

$$\frac{1276}{1000000} = 0.001276$$

$$(21)^2 = 441$$

$$60 = 1$$

$$70 = 1$$

6000 160000

$$11^2 = 121 \quad (a+b)^2 = a^2 + 2ab + b^2$$

$$11^3 = 1331 \quad (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$x^2 + 3x + 2 = (x+1)(x+2)$$

$$x^2 + 4x - 5 = (x+5)(x-1)$$

$$x^2 - 7x - 8 = (x-8)(x+1)$$

$$\frac{1}{2} + \frac{3}{2} - \frac{5}{2} = \frac{-1}{2} \quad \frac{3}{4} + \frac{2}{6} - \frac{5}{2} = \frac{9+4-30}{12} = \frac{-17}{12} = -1\frac{5}{12}$$

$$(98)_{ba}^2 = 9604 \quad \begin{array}{|c|c|} \hline 6 & 15 \\ \hline +144 & +81 \\ \hline 150 & 96 \\ \hline \end{array}$$

$$(23)^2 = 529 \quad \begin{array}{|c|} \hline 1 \\ \hline +4 \\ \hline 5 \\ \hline \end{array}$$

$$ax^2 + bx + c$$

$$ac + \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$11^2 = 121 \quad (a+b)^2 = a^2 + 2ab + b^2$$

$$(98)_{ba}^2 = 9604$$

$$\begin{array}{|c|c|} \hline 6 & 15 \\ \hline +144 & +81 \\ \hline 150 & 96 \\ \hline \end{array}$$

$$(23)^2 = 529$$

$$x^3 - 3x^2y + 3xy^2 - y^3$$

સ્વાધ્યાય [પાઠ નંબર 32-33]

ખાલી જગ્યા પૂરો :

(1) $(-51)^0 = \underline{1}$.

કારણ : શૂન્ય સિવાયની કોઈ પણ સંખ્યાનો ઘાતાંક શૂન્ય હોય, તો તેની કિંમત 1 થાય. $a \neq 0$ માટે $a^0 = 1$

(2) $x^5 \times x^{-4} \div x^2 = \underline{\frac{1}{x}}$

રીત : $x^5 \times x^{-4} \div x^2 = x^5 \times x^{-4} \times x^{-2}$
 $= x^{5+(-4)+(-2)}$
 $= x^{5+(-6)}$
 $= x^{-1}$
 $= \frac{1}{x}$

(3) $(a^3)^{-4} = \underline{\frac{1}{a^{12}}}$

રીત : $(a^3)^{-4} = a^{3 \times (-4)} = a^{-12} = \frac{1}{a^{12}}$

(4) $(\sqrt{y})^5 = \underline{y^{\frac{5}{2}}}$

રીત : $(\sqrt{y})^5 = (y^{\frac{1}{2}})^5 = y^{\frac{1}{2} \times 5} = y^{\frac{5}{2}}$

(5) $4^{-2} \times \frac{1}{4^{-2}} = \underline{1}$

રીત : $4^{-2} \times \frac{1}{4^{-2}} = 4^{-2} \times 4^2 = 4^{(-2)+2} = 4^0 = 1$

(6) $\frac{1}{(3 \times 4)^{-1}} = \underline{12}$

રીત : $\frac{1}{(3 \times 4)^{-1}} = \frac{1}{(12)^{-1}} = 12^1 = 12$

(7) $\left[\left(\frac{2}{3}\right)^2\right]^{-2} = \underline{5 \frac{1}{16}}$

રીત : $\left[\left(\frac{2}{3}\right)^2\right]^{-2} = \left(\frac{2}{3}\right)^{2 \times (-2)}$
 $= \left(\frac{2}{3}\right)^{(-4)} = \left(\frac{3}{2}\right)^4 = \frac{81}{16} = 5 \frac{1}{16}$

(8) $\left[\frac{16}{81}\right]^{\frac{1}{4}} = \underline{\frac{2}{3}}$

રીત : $\left[\frac{16}{81}\right]^{\frac{1}{4}} = \left[\frac{2^4}{3^4}\right]^{\frac{1}{4}}$
 $= \frac{2^{4 \times \frac{1}{4}}}{3^{4 \times \frac{1}{4}}}$
 $= \frac{2}{3}$

2. સાદું રૂપ આપો :

(1) $\left(\frac{x^2}{x^3}\right)^2 \cdot \left(\frac{x^3}{x^4}\right)^3 \cdot \left(\frac{x^4}{x^2}\right)^4$; (જ્યાં $x > 0$)

$= \left(\frac{x^2}{x^3}\right)^2 \cdot \left(\frac{x^3}{x^4}\right)^3 \cdot \left(\frac{x^4}{x^2}\right)^4$

$= \frac{x^2}{x^3} \times \frac{x^3}{x^4} \times \frac{x^4}{x^2}$

$= \frac{x^1}{x^3} \times \frac{x^1}{x^4} \times \frac{x^1}{x^2}$

$= x^{1-\frac{2}{3}} \times x^{1-\frac{3}{4}} \times x^{1-2}$

$$\begin{aligned}
 &= \frac{(3^3 \times 2^2)^{\frac{1}{3}} \times (2^2)^{\frac{1}{6}}}{(3^4)^{\frac{1}{4}}} \\
 &= \frac{3^{\frac{3}{3}} \times 2^{\frac{2}{3}} \times 2^{\frac{2}{6}}}{3^{4 \times \frac{1}{4}}} \\
 &= \frac{3^1 \times 2^{\frac{2}{3}} \times 2^{\frac{1}{3}}}{3^1} \\
 &= 3^{1-1} \times 2^{\frac{2}{3}} \times 2^{\frac{1}{3}} \\
 &= 3^0 \times 2^{\frac{2}{3} + \frac{1}{3}} \\
 &= 1 \times 2^{\frac{2+1}{3}} \\
 &= 1 \times 2^{\frac{3}{3}} \\
 &= 1 \times 2^1 \\
 &= 2
 \end{aligned}$$

(4) $\left(\frac{8}{27}\right)^{\frac{1}{3}} \times \left(\frac{9}{25}\right)^{\frac{1}{2}} \times \left(\frac{2}{5}\right)^{-1}$

$$\begin{aligned}
 &= \left(\frac{2^3}{3^3}\right)^{\frac{1}{3}} \times \left(\frac{3^2}{5^2}\right)^{\frac{1}{2}} \times \left(\frac{5}{2}\right)^1 \\
 &= \frac{2^{\frac{3}{3}}}{3^{\frac{3}{3}}} \times \frac{3^{\frac{2}{2}}}{5^{\frac{2}{2}}} \times \frac{5}{2} \\
 &= \frac{2}{3} \times \frac{3}{5} \times \frac{5}{2} \\
 &= \frac{2 \times 3 \times 5}{2 \times 3 \times 5} \\
 &= 2^{1-1} \times 3^{1-1} \times 5^{1-1} \\
 &= 2^0 \times 3^0 \times 5^0 \\
 &= 1 \times 1 \times 1 \\
 &= 1
 \end{aligned}$$

4. साभित करो : $\left(\frac{\frac{1}{2^3}}{2^{\frac{-1}{3}}}\right)^3 + \frac{3^{\frac{1}{2}}}{3^{\frac{-1}{2}}} = 7$

अ.अ. = $\left(\frac{2^{\frac{1}{3}}}{2^{\frac{-1}{3}}}\right)^3 + \frac{3^{\frac{1}{2}}}{3^{\frac{-1}{2}}}$

$$\begin{aligned}
 &= \left(2^{\frac{1}{3} - \left(\frac{-1}{3}\right)}\right)^3 + 3^{\frac{1}{2} - \left(\frac{-1}{2}\right)} \\
 &= \left(2^{\frac{2}{3}}\right)^3 + 3^{\frac{1}{2} + \frac{1}{2}} \\
 &= 2^{\frac{2}{3} \times 3} + 3^2 \\
 &= 2^2 + 3^1 \\
 &= 4 + 3 \\
 &= 7 \\
 &= \text{अ.अ.}
 \end{aligned}$$

5. $\frac{(16)^{\frac{1}{4}}}{(27)^{\frac{1}{3}}} + \frac{(625)^{\frac{1}{4}}}{(81)^{\frac{1}{4}}} - \frac{1}{(243)^{\frac{1}{5}}} = 2$ साभित करो.

अ.अ. = $\frac{(16)^{\frac{1}{4}}}{(27)^{\frac{1}{3}}} + \frac{(625)^{\frac{1}{4}}}{(81)^{\frac{1}{4}}} - \frac{1}{(243)^{\frac{1}{5}}}$

$$\begin{aligned}
 &= \frac{(2^4)^{\frac{1}{4}}}{(3^3)^{\frac{1}{3}}} + \frac{(5^4)^{\frac{1}{4}}}{(3^4)^{\frac{1}{4}}} - \frac{1}{(3^5)^{\frac{1}{5}}} \\
 &= \frac{2^{\frac{4}{4}}}{3^{\frac{3}{3}}} + \frac{5^{\frac{4}{4}}}{3^{\frac{4}{4}}} - \frac{1}{3^{\frac{5}{5}}} \\
 &= \frac{2^1}{3^1} + \frac{5^1}{3^1} - \frac{1}{3^1} \\
 &= \frac{2}{3} + \frac{5}{3} - \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned} &= \frac{2+5-1}{3} \\ &= \frac{6}{3} \\ &= 2 \\ &= \text{જ.બા.} \end{aligned}$$

સાબિત કરો : $[(a^x)^y \cdot (a^y)^x]^z = a^{2xyz}$ ($x, y, z \in \mathbb{Q}$)

$$\begin{aligned} \text{ડા.બા.} &= [(a^x)^y \cdot (a^y)^x]^z \\ &= [a^{xy} \cdot a^{yx}]^z \\ &= [a^{xy+yx}]^z \\ &= [a^{2xy}]^z \\ &= a^{2xy \times z} \\ &= a^{2xyz} \\ &= \text{જ.બા.} \end{aligned}$$

7. જો $x > 0$ તથા $x \in \mathbb{Q}$ અને a, b, c શૂન્યેતર હોય, તો સાબિત કરો

$$\left[\left(\frac{x^a}{x^b} \right)^{\frac{1}{a}} \right]^{\frac{1}{b}} \cdot \left[\left(\frac{x^b}{x^c} \right)^{\frac{1}{b}} \right]^{\frac{1}{c}} \cdot \left[\left(\frac{x^c}{x^a} \right)^{\frac{1}{c}} \right]^{\frac{1}{a}} = 1$$

$$\begin{aligned} \text{ડા.બા.} &= \left[\left(\frac{x^a}{x^b} \right)^{\frac{1}{a}} \right]^{\frac{1}{b}} \cdot \left[\left(\frac{x^b}{x^c} \right)^{\frac{1}{b}} \right]^{\frac{1}{c}} \cdot \left[\left(\frac{x^c}{x^a} \right)^{\frac{1}{c}} \right]^{\frac{1}{a}} \\ &= \left[\frac{x^{a \times \frac{1}{a}}}{x^{b \times \frac{1}{a}}} \right]^{\frac{1}{b}} \cdot \left[\frac{x^{b \times \frac{1}{b}}}{x^{c \times \frac{1}{b}}} \right]^{\frac{1}{c}} \cdot \left[\frac{x^{c \times \frac{1}{c}}}{x^{a \times \frac{1}{c}}} \right]^{\frac{1}{a}} \\ &= \left[\frac{x^1}{x^{\frac{b}{a}}} \right]^{\frac{1}{b}} \cdot \left[\frac{x^1}{x^{\frac{c}{b}}} \right]^{\frac{1}{c}} \cdot \left[\frac{x^1}{x^{\frac{a}{c}}} \right]^{\frac{1}{a}} \end{aligned}$$

$$\begin{aligned} &= \left[\frac{x^{\frac{1 \times 1}{b}}}{x^{a \times \frac{1}{b}}} \right] \cdot \left[\frac{x^{\frac{1 \times 1}{c}}}{x^{b \times \frac{1}{c}}} \right] \cdot \left[\frac{x^{\frac{1 \times 1}{a}}}{x^{c \times \frac{1}{a}}} \right] \\ &= \frac{x^{\frac{1}{b}}}{x^{\frac{a}{b}}} \times \frac{x^{\frac{1}{c}}}{x^{\frac{b}{c}}} \times \frac{x^{\frac{1}{a}}}{x^{\frac{c}{a}}} \\ &= \frac{x^{\frac{1}{a}}}{x^{\frac{a}{a}}} \times \frac{x^{\frac{1}{b}}}{x^{\frac{b}{b}}} \times \frac{x^{\frac{1}{c}}}{x^{\frac{c}{c}}} \\ &= x^{\frac{1}{a} - \frac{1}{a}} \times x^{\frac{1}{b} - \frac{1}{b}} \times x^{\frac{1}{c} - \frac{1}{c}} \\ &= x^0 \times x^0 \times x^0 \\ &= 1 \times 1 \times 1 \\ &= 1 \\ &= \text{જ.બા.} \end{aligned}$$

પ્રવૃત્તિ

* કહો જોઈએ :

(1) 365 એ કઈ બે ક્રમિક સંખ્યાઓના વર્ગના સરવાળા બરાબર છે?

(2) 365 એ કઈ ત્રણ ક્રમિક સંખ્યાઓના વર્ગના સરવાળા બરાબર છે?

જવાબ : (1) $365 = 13^2 + 14^2$ (2) $365 = 10^2 + 11^2 + 12^2$

(4) 24×19

$$\begin{aligned}
 &= (20 + 4)(20 - 1) \\
 &= (20)^2 + (4 - 1)(20) + (4)(-1) \\
 &= 400 + (3)(20) - 4 \\
 &= 400 + 60 - 4 \\
 &= 456
 \end{aligned}$$

(5) 23×18

$$\begin{aligned}
 &= (20 + 3)(20 - 2) \\
 &= (20)^2 + (3 - 2)(20) + (3)(-2) \\
 &= 400 + (1)(20) - 6 \\
 &= 400 + 20 - 6 \\
 &= 414
 \end{aligned}$$

(6) 27×32

$$\begin{aligned}
 &= (30 - 3)(30 + 2) \\
 &= (30)^2 + (-3 + 2)(30) + (-3)(2) \\
 &= 900 + (-1)(30) - 6 \\
 &= 900 - 30 - 6 \\
 &= 864
 \end{aligned}$$

स्वाध्याय [पान नंबर 57-58]

1. विस्तरण करो :

(1) $2a(-3a^2)$

$$= -6a^3$$

(2) $(-4ab)(6a^2b)$

$$= -24a^3b^2$$

(3) $(2m)(3m + n)$

$$\begin{aligned}
 &= (2m \times 3m) + (2m \times n) \\
 &= 6m^2 + 2mn
 \end{aligned}$$

(4) $(-4n)(6n + 5m)$

$$\begin{aligned}
 &= (-4n \times 6n) + (-4n \times 5m) \\
 &= -24n^2 - 20mn
 \end{aligned}$$

(5) $(5a + 3b)(6a - 2b)$

$$\begin{aligned}
 &= 5a(6a - 2b) + 3b(6a - 2b) \\
 &= 30a^2 - 10ab + 18ab - 6b^2 \\
 &= 30a^2 + 8ab - 6b^2
 \end{aligned}$$

(6) $(2x + 3y)(6x - 2y)$

$$\begin{aligned}
 &= 2x(6x - 2y) + 3y(6x - 2y) \\
 &= 12x^2 - 4xy + 18xy - 6y^2 \\
 &= 12x^2 + 14xy - 6y^2
 \end{aligned}$$

(7) $(6xy + 1)(2xy - 3)$

$$\begin{aligned}
 &= 6xy(2xy - 3) + 1(2xy - 3) \\
 &= 12x^2y^2 - 18xy + 2xy - 3 \\
 &= 12x^2y^2 - 16xy - 3
 \end{aligned}$$

(8) $(a - 2b)(2a - b)$

$$\begin{aligned}
 &= a(2a - b) - 2b(2a - b) \\
 &= 2a^2 - ab - 4ab + 2b^2 \\
 &= 2a^2 - 5ab + 2b^2
 \end{aligned}$$

2. विस्तरण करो :

(1) $(a + 5)^2$

$$\begin{aligned}
 &= (a)^2 + 2(a)(5) + (5)^2 \\
 &= a^2 + 10a + 25
 \end{aligned}$$

(2) $(m - 7)^2$

$$\begin{aligned}
 &= (m)^2 - 2(m)(7) + (7)^2 \\
 &= m^2 - 14m + 49
 \end{aligned}$$

(3) $(3m + 2n)^2$

$$\begin{aligned}
 &= (3m)^2 + 2(3m)(2n) + (2n)^2 \\
 &= 9m^2 + 12mn + 4n^2
 \end{aligned}$$

(4) $(4xy - 3)^2$

$$\begin{aligned}
 &= (4xy)^2 - 2(4xy)(3) + (3)^2 \\
 &= 16x^2y^2 - 24xy + 9
 \end{aligned}$$

$$\begin{aligned} &) (5ab + 3c)^2 \\ & = (5ab)^2 + 2(5ab)(3c) + (3c)^2 \\ & = 25a^2b^2 + 30abc + 9c^2 \end{aligned}$$

$$\begin{aligned} &) (4b^2 + 3)^2 \\ & = (4b^2)^2 + 2(4b^2)(3) + (3)^2 \\ & = 16b^4 + 24b^2 + 9 \end{aligned}$$

$$\begin{aligned} &) (2x - 7)^2 \\ & = (2x)^2 - 2(2x)(7) + (7)^2 \\ & = 4x^2 - 28x + 49 \end{aligned}$$

$$\begin{aligned} &) (5 - 3mn)^2 \\ & = (5)^2 - 2(5)(3mn) + (3mn)^2 \\ & = 25 - 30mn + 9m^2n^2 \end{aligned}$$

विस्तरण करो :

$$\begin{aligned} &) (x - 7)(x + 7) \\ & = (x)^2 - (7)^2 \\ & = x^2 - 49 \end{aligned}$$

$$\begin{aligned} & (2) (2a + 3b)(2a - 3b) \\ & = (2a)^2 - (3b)^2 \\ & = 4a^2 - 9b^2 \end{aligned}$$

$$\begin{aligned} &) (2m + 5)(2m - 5) \\ & = (2m)^2 - (5)^2 \\ & = 4m^2 - 25 \end{aligned}$$

$$\begin{aligned} & (4) (2mn + 3)(2mn - 3) \\ & = (2mn)^2 - (3)^2 \\ & = 4m^2n^2 - 9 \end{aligned}$$

विस्तरण करो :

$$\begin{aligned} & 1) (a + 3)(a + 2) \\ & = (a)^2 + (3 + 2)(a) + (3)(2) \\ & = a^2 + (5)(a) + 6 \\ & = a^2 + 5a + 6 \end{aligned}$$

$$\begin{aligned} & 2) (m - 2)(m - 5) \\ & = (m)^2 + (-2 - 5)(m) + (-2)(-5) \\ & = m^2 + (-7)(m) + 10 \\ & = m^2 - 7m + 10 \end{aligned}$$

$$\begin{aligned} & (3) (x - 9)(x + 2) \\ & = (x)^2 + (-9 + 2)(x) + (-9)(2) \\ & = x^2 + (-7)(x) - 18 \\ & = x^2 - 7x - 18 \end{aligned}$$

$$\begin{aligned} & (4) (x + 6y)(x - 2y) \\ & = (x)^2 + (6y - 2y)(x) + (6y)(-2y) \\ & = x^2 + (4y)(x) - 12y^2 \\ & = x^2 + 4xy - 12y^2 \end{aligned}$$

$$\begin{aligned} & (5) (5x - 2y)(5x - 4y) \\ & = (5x)^2 + (-2y - 4y)(5x) + (-2y)(-4y) \\ & = 25x^2 + (-6y)(5x) + 8y^2 \\ & = 25x^2 - 30xy + 8y^2 \end{aligned}$$

$$\begin{aligned} & (6) (2m + 3n)(2m + 5n) \\ & = (2m)^2 + (3n + 5n)(2m) + (3n)(5n) \\ & = 4m^2 + (8n)(2m) + 15n^2 \\ & = 4m^2 + 16mn + 15n^2 \end{aligned}$$

$$\begin{aligned} & (7) (xy - 7)(xy + 4) \\ & = (xy)^2 + (-7 + 4)(xy) + (-7)(4) \\ & = x^2y^2 + (-3)(xy) - 28 \\ & = x^2y^2 - 3xy - 28 \end{aligned}$$

$$\begin{aligned} & (8) (x^2 - 5)(x^2 + 3) \\ & = (x^2)^2 + (-5 + 3)(x^2) + (-5)(3) \\ & = x^4 + (-2)(x^2) - 15 \\ & = x^4 - 2x^2 - 15 \end{aligned}$$

5. विस्तरण सूत्रको उपयोग करी किंमत मेणवो :

$$\begin{aligned} & (1) 62^2 \\ & = (60 + 2)^2 \\ & = (60)^2 + 2(60)(2) + (2)^2 \\ & = 3600 + 240 + 4 \\ & = 3844 \end{aligned}$$

$$\begin{aligned} & (2) 57^2 \\ & = (60 - 3)^2 \\ & = (60)^2 - 2(60)(3) + (3)^2 \\ & = 3600 - 360 + 9 \\ & = 3249 \end{aligned}$$

અવયવો પાડો :

(1) $x^2 + 12x + 36$

$$= (x)^2 + 2(x)(6) + (6)^2$$

$$= (x + 6)^2$$

(2) $4x^2 + 12xy + 9y^2$

$$= (2x)^2 + 2(2x)(3y) + (3y)^2$$

$$= (2x + 3y)^2$$

(3) $9x^2 + 48x + 64$

$$= (3x)^2 + 2(3x)(8) + (8)^2$$

$$= (3x + 8)^2$$

(4) $x^2 - 8x + 16$

$$= (x)^2 - 2(x)(4) + (4)^2$$

$$= (x - 4)^2$$

(5) $25x^2y^2 - 20xy + 4$

$$= (5xy)^2 - 2(5xy)(2) + (2)^2$$

$$= (5xy - 2)^2$$

(6) $16x^2 + 40x + 25$

$$= (4x)^2 + 2(4x)(5) + (5)^2$$

$$= (4x + 5)^2$$

(7) $81 - 90xy + 25x^2y^2$

$$= (9)^2 - 2(9)(5xy) + (5xy)^2$$

$$= (9 - 5xy)^2$$

(8) $3x^3 - 30x^2 + 75x$

$$= 3x(x^2 - 10x + 25)$$

$$= 3x\{(x)^2 - 2(x)(5) + (5)^2\}$$

$$= 3x(x - 5)^2$$

નવું શીખીએ ... યાદ રાખીએ

* ત્રિપદીના વર્ગનું વિસ્તરણ :

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

આ વિસ્તરણ સૂત્રની બાજુઓ ઉલટાવવાથી નીચેનું અવયવ સૂત્ર મળે છે :

$$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$$

* અવયવ સૂત્રમાં -

ભાગ 1 : સરવાળાથી જોડાયેલાં ત્રણ પૂર્ણવર્ગ પદો છે.

ભાગ 2 : પહેલા અને બીજા પદનાં વર્ગમૂળોનો ગુણાકાર $\times 2$ બીજા અને ત્રીજા પદનાં વર્ગમૂળોનો ગુણાકાર $\times 2$ ત્રીજા અને પહેલા પદનાં વર્ગમૂળોનો ગુણાકાર $\times 2$

* ત્રિપદીના વર્ગના વિસ્તરણથી મળતાં બધાં જ પદો ધન હોય અથવા પૂર્ણવર્ગ સિવાયનાં પદોમાં બે અને માત્ર બે જ પદો ઋણ હોય.

મહાવરો 5 [પાન નંબર 44 - 45]

અવયવો પાડો :

(1) $9x^2 + 4y^2 + 1 + 12xy + 4y + 6x$

$$= (3x)^2 + (2y)^2 + (1)^2 + 2(3x)(2y) + 2(2y)(1) + 2(1)(3x)$$

$$= (3x + 2y + 1)^2$$

(2) $16a^2 + 9b^2 + c^2 - 24ab + 6bc - 8ca$

$$= (4a)^2 + (-3b)^2 + (-c)^2 + 2(4a)(-3b)$$

$$+ 2(-3b)(-c) + 2(-c)(4a)$$

$$= (4a - 3b - c)^2$$

અથવા

$$16a^2 + 9b^2 + c^2 - 24ab + 6bc - 8ca$$

$$= (-4a)^2 + (3b)^2 + (c)^2$$

$$+ 2(-4a)(3b) + 2(3b)(c) + 2(c)(-4a)$$

$$= (-4a + 3b + c)^2$$

$$(3) a^4 + 4b^2 + 9 + 4a^2b - 12b - 6a^2$$

$$= (a^2)^2 + (2b)^2 + (-3)^2 + 2(a^2)(2b) + 2(2b)(-3) + 2(-3)(a^2)$$

$$= (a^2 + 2b - 3)^2$$

અથવા $(-a^2 - 2b + 3)^2$ જવાબ પણ મળે.

$$(4) 9x^2 + 16y^2 + 25 + 24xy - 40y - 30x$$

$$= (3x)^2 + (4y)^2 + (-5)^2 + 2(3x)(4y) + 2(4y)(-5) + 2(-5)(3x)$$

$$= (3x + 4y - 5)^2$$

અથવા $(-3x - 4y + 5)^2$ જવાબ પણ મળે.

$$(5) a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ca$$

$$= (a)^2 + (-2b)^2 + (c)^2 + 2(a)(-2b) + 2(-2b)(c) + 2(c)(a)$$

$$= (a - 2b + c)^2$$

અથવા $(-a + 2b - c)^2$ જવાબ પણ મળે.

સ્વાધ્યાય [પાન નંબર 45-46]

1. નીચેનું પ્રત્યેક વિધાન સાચું બને એ રીતે ખાલી જગ્યા પૂરો :

$$(1) 15x^3y = 3 \times 5 \times x \times x \times x \times y$$

$$(2) 5x^4 - x^3 = x^3(5x - 1)$$

$$(3) -5a^2 + 10a = -5a(a - 2)$$

$$(4) ab + a - 2b - 2 = (a - 2)(b + 1)$$

$$\text{રીત : } ab + a - 2b - 2 = a(b + 1) - 2(b + 1)$$

$$= (b + 1)(a - 2)$$

$$(5) 16a^2 + 8a + 1 = (4a + 1)^2$$

$$\text{રીત : મ. પ.} = + 2 \times \sqrt{\text{પ્ર. પ.}} \times \sqrt{\text{અં. પ.}}$$

$$= + 2 \times \sqrt{16a^2} \times \sqrt{1}$$

$$= + 2 \times 4a \times 1 = 8a$$

$$(6) x^2 + 10x + 25 = (x + 5)^2$$

$$\text{રીત : મ. પ.} = \frac{(\text{મ. પ.})^2}{4 \times (\text{અં. પ.})}$$

$$= \frac{(10x)^2}{4 \times 25} = \frac{10x \times 10x}{4 \times 25} = x^2$$

$$(7) 4y^2 - 12y + 9 = (2y - 3)^2$$

$$\text{રીત : મ. પ.} = - 2 \times \sqrt{\text{પ્ર. પ.}} \times \sqrt{\text{અં. પ.}}$$

$$= - 2 \times \sqrt{4y^2} \times \sqrt{9}$$

$$= - 2 \times 2y \times 3 = - 12y$$

$$(8) 16x^2 - 72x + 81 \text{ એ } 4x - 9 \text{ નો વર્ગ છે.}$$

$$\text{રીત : } 16x^2 - 72x + 81 = (4x)^2 - 2(4x)(9) + (9)^2$$

$$= (4x - 9)^2$$

$$(9) a^2 - 0.4a + 0.04 = (a - 0.2)^2$$

$$\text{રીત : મ. પ.} = - 2 \times \sqrt{\text{પ્ર. પ.}} \times \sqrt{\text{અં. પ.}}$$

$$= - 2 \times \sqrt{a^2} \times \sqrt{0.04}$$

$$= - 2 \times a \times 0.2 = - 0.4a$$

$$(10) 9x^2 + 1 \text{ માં } \pm 6x \text{ ઉમેરતાં પૂર્ણવર્ગ ત્રિપદી બને.}$$

$$\text{રીત : } 9x^2 \text{ જે } (3x)^2 \text{ છે. } 1 \text{ જે } (1)^2 \text{ છે.}$$

આમ, $9x^2$ એ પ્ર. પ. અને 1 એ અં. પ. હોઈ શકે.

$$\text{મ. પ.} = \pm 2 \times \sqrt{\text{પ્ર. પ.}} \times \sqrt{\text{અં. પ.}}$$

$$= \pm 2 \times \sqrt{9x^2} \times \sqrt{1}$$

$$= \pm 2 \times 3x \times 1 = \pm 6x$$

$9x^2 + 1$ માં મ. પ. $\pm 6x$ ઉમેરતાં પૂર્ણવર્ગ ત્રિપદી બને.

2. અવયવો પાડો :

$$(1) 4ab + 8a - b - 2$$

$$= 4ab + 8a - b - 2$$

$$= 4a(b + 2) - 1(b + 2)$$

$$= (b + 2)(4a - 1)$$

$$(2) x^2y - 3x^2 + y - 3$$

$$= x^2y - 3x^2 + y - 3$$

$$= x^2(y - 3) + 1(y - 3)$$

$$= (y - 3)(x^2 + 1)$$

$$\begin{aligned} (3) \quad 2x^2 - 5a - 5x + 2ax &= 2x^2 + 2ax - 5x - 5a \\ &= 2x(x+a) - 5(x+a) \\ &= (x+a)(2x-5) \end{aligned}$$

$$\begin{aligned} (4) \quad 3ab + 12 - 4a - 9b &= 3ab - 4a - 9b + 12 \\ &= a(3b-4) - 3(3b-4) \\ &= (3b-4)(a-3) \end{aligned}$$

$$\begin{aligned} (5) \quad x^2 + 49 + 14x &= x^2 + 14x + 49 \\ &= (x)^2 + 2(x)(7) + (7)^2 \\ &= (x+7)^2 \end{aligned}$$

$$\begin{aligned} (6) \quad 16a^2 + 40ab + 25b^2 &= (4a)^2 + 2(4a)(5b) + (5b)^2 \\ &= (4a+5b)^2 \end{aligned}$$

$$\begin{aligned} (7) \quad m^4 - 16m^2 + 64 &= (m^2)^2 - 2(m^2)(8) + (8)^2 \\ &= (m^2-8)^2 \end{aligned}$$

$$\begin{aligned} (8) \quad 4y^3 - 28y^2 + 49y &= y(4y^2 - 28y + 49) \\ &= y\{(2y)^2 - 2(2y)(7) + (7)^2\} \\ &= y(2y-7)^2 \end{aligned}$$

$$\begin{aligned} (9) \quad 25x^2 + 4y^2 + 9z^2 + 20xy + 12yz + 30zx &= (5x)^2 + (2y)^2 + (3z)^2 + 2(5x)(2y) + 2(2y)(3z) \\ &\quad + 2(3z)(5x) \\ &= (5x+2y+3z)^2 \end{aligned}$$

$$\begin{aligned} (10) \quad 4m^2 + 9n^2 + p^2 - 12mn + 6np - 4pm &= (2m)^2 + (-3n)^2 + (-p)^2 + 2(2m)(-3n) + 2(-3n)(-p) \\ &\quad + 2(-p)(2m) \\ &= (2m-3n-p)^2 \end{aligned}$$

અથવા $(-2m+3n+p)^2$ જવાબ પણ મળે.

પ્રવૃત્તિ

જેના અવયવ પરે તેવી ઉપરની રીતોવાળી પાંચ બહુપદીઓ અલગ અલગ કાગળમાં લખી ચિટ્ટીઓ બનાવો. આ ચિટ્ટીઓ ઉછાળી જેના ભાગમાં જે ચિટ્ટી આવે, તે મિત્ર તે બહુપદીના અવયવો પાડે. આવી જુદી જુદી બહુપદીઓ લખી અથવા ચિટ્ટીઓ ક્રમાનુસાર ફેરવીને આ રમત પાંચ વખત રમો.

જાણો :

$$(x-y)^0 = 1$$

$$(x-y)^1 = x-y$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x-y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$$

$$(x-y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$$

જુઓ, x ની ઘાત ઊતરતા ક્રમમાં છે જ્યારે y ની ઘાત ચડતા ક્રમમાં છે.

$$\begin{aligned}
&= (x)^2 - (y)^2 \quad (2a + 3 = x \text{ અને } m - n = y \text{ ધારતાં}) \\
&= (x + y)(x - y) \\
&= [(2a + 3) + (m - n)][(2a + 3) - (m - n)] \\
&\quad (x = 2a + 3 \text{ અને } y = m - n \text{ મૂકતાં}) \\
&= (2a + 3 + m - n)(2a + 3 - m + n)
\end{aligned}$$

$$\begin{aligned}
(4) \quad &x^2 - 14x + 49 - a^2 - 12a - 36 \\
&= (x^2 - 14x + 49) - (a^2 + 12a + 36) \\
&= (x - 7)^2 - (a + 6)^2 \\
&= (m)^2 - (n)^2 \quad (x - 7 = m \text{ અને } a + 6 = n \text{ ધારતાં}) \\
&= (m + n)(m - n) \\
&= [(x - 7) + (a + 6)][(x - 7) - (a + 6)] \\
&\quad (m = x - 7 \text{ અને } n = a + 6 \text{ મૂકતાં}) \\
&= (x - 7 + a + 6)(x - 7 - a - 6) \\
&= (x + a - 1)(x - a - 13)
\end{aligned}$$

નવું શીખીએ ... યાદ રાખીએ

$x^4 + ky^4$ સ્વરૂપની બહુપદીના અવયવો પાડવા :

દા. ત., $x^4 + 4$ ના અવયવો પાડો.

$$\begin{aligned}
\text{મધ્યમ પદ} &= \pm 2 \times \sqrt{x^4} \times \sqrt{4} \\
&= \pm 2 \times x^2 \times 2 = \pm 4x^2 \\
x^4 + 4 &= x^4 + 4x^2 + 4 - 4x^2 \\
&= (x^2 + 2)^2 - (2x)^2 \\
&= (m)^2 - (2x)^2 \quad (x^2 + 2 = m \text{ ધારતાં}) \\
&= (m + 2x)(m - 2x) \\
&= [(x^2 + 2) + 2x][(x^2 + 2) - 2x] \\
&\quad (m = x^2 + 2 \text{ મૂકતાં}) \\
&= (x^2 + 2 + 2x)(x^2 + 2 - 2x) \\
&= (x^2 + 2x + 2)(x^2 - 2x + 2)
\end{aligned}$$

મહાવરો 3 [પાઠ નંબર 54]

અવયવ પાડો :

$$\begin{aligned}
(1) \quad &4x^4 + y^4 \\
\text{મધ્યમ પદ} &= \pm 2 \times \sqrt{4x^4} \times \sqrt{y^4} \\
&= \pm 2 \times 2x^2 \times y^2 = \pm 4x^2y^2 \\
4x^4 + y^4 &= 4x^4 + 4x^2y^2 + y^4 - 4x^2y^2 \\
&= (4x^4 + 4x^2y^2 + y^4) - (4x^2y^2) \\
&= (2x^2 + y^2)^2 - (2xy)^2 \\
&= (m)^2 - (2xy)^2 \quad (2x^2 + y^2 = m \text{ ધારતાં}) \\
&= (m + 2xy)(m - 2xy) \\
&= [(2x^2 + y^2) + 2xy][(2x^2 + y^2) - 2xy] \\
&\quad (m = 2x^2 + y^2 \text{ મૂકતાં}) \\
&= (2x^2 + y^2 + 2xy)(2x^2 + y^2 - 2xy)
\end{aligned}$$

(2) $64a^4 + b^4$

$$\begin{aligned}
\text{મધ્યમ પદ} &= \pm 2 \times \sqrt{64a^4} \times \sqrt{b^4} \\
&= \pm 2 \times 8a^2 \times b^2 = \pm 16a^2b^2 \\
64a^4 + b^4 &= 64a^4 + 16a^2b^2 + b^4 - 16a^2b^2 \\
&= (64a^4 + 16a^2b^2 + b^4) - (16a^2b^2) \\
&= (8a^2 + b^2)^2 - (4ab)^2 \\
&= (m)^2 - (4ab)^2 \quad (8a^2 + b^2 = m \text{ ધારતાં}) \\
&= (m + 4ab)(m - 4ab) \\
&= [(8a^2 + b^2) + 4ab][(8a^2 + b^2) - 4ab] \\
&\quad (m = 8a^2 + b^2 \text{ મૂકતાં}) \\
&= (8a^2 + b^2 + 4ab)(8a^2 + b^2 - 4ab) \\
&= (8a^2 + 4ab + b^2)(8a^2 - 4ab + b^2)
\end{aligned}$$

(3) $81a^4 + 4$

$$\begin{aligned}
\text{મધ્યમ પદ} &= \pm 2 \times \sqrt{81a^4} \times \sqrt{4} \\
&= \pm 2 \times 9a^2 \times 2 = \pm 36a^2 \\
81a^4 + 4 &= 81a^4 + 36a^2 + 4 - 36a^2 \\
&= (81a^4 + 36a^2 + 4) - (36a^2) \\
&= (9a^2 + 2)^2 - (6a)^2
\end{aligned}$$

$$\begin{aligned}
 &= (m)^2 - (6a)^2 \quad (9a^2 + 2 = m \text{ ધારતી}) \\
 &= (m + 6a)(m - 6a) \\
 &= [(9a^2 + 2) + 6a][(9a^2 + 2) - 6a] \\
 &\quad (m = 9a^2 + 2 \text{ મૂકતી}) \\
 &= (9a^2 + 2 + 6a)(9a^2 + 2 - 6a) \\
 &= (9a^2 + 6a + 2)(9a^2 - 6a + 2)
 \end{aligned}$$

(4) $4m^4 + 625$

$$\begin{aligned}
 \text{મધ્યમ પદ} &= \pm 2 \times \sqrt{4m^4} \times \sqrt{625} \\
 &= \pm 2 \times 2m^2 \times 25 = \pm 100m^2 \\
 4m^4 + 625 &= 4m^4 + 100m^2 + 625 - 100m^2 \\
 &= (4m^4 + 100m^2 + 625) - (100m^2) \\
 &= (2m^2 + 25)^2 - (10m)^2 \\
 &= (a)^2 - (10m)^2 \quad (2m^2 + 25 = a \text{ ધારતી}) \\
 &= (a + 10m)(a - 10m) \\
 &= [(2m^2 + 25) + 10m][(2m^2 + 25) - 10m] \\
 &\quad (a = 2m^2 + 25 \text{ મૂકતી}) \\
 &= (2m^2 + 25 + 10m)(2m^2 + 25 - 10m) \\
 &= (2m^2 + 10m + 25)(2m^2 - 10m + 25)
 \end{aligned}$$

નવું શીખીએ ... યાદ રાખીએ

$x^4 + ax^2 + 1$ સ્વરૂપની બહુપદીના અવયવો પાડવા :

દા. ત., $x^4 + x^2 + 1$ ના અવયવો પાડો.

$$\begin{aligned}
 \text{મધ્યમ પદ} &= \pm 2 \times \sqrt{x^4} \times \sqrt{1} \\
 &= \pm 2 \times x^2 \times 1 = \pm 2x^2 \\
 x^4 + x^2 + 1 &= x^4 + 2x^2 + 1 - x^2 \\
 &= (x^2 + 1)^2 - (x)^2 \\
 &= (m)^2 - (x)^2 \quad (x^2 + 1 = m \text{ ધારતી}) \\
 &= (m + x)(m - x) \\
 &= [(x^2 + 1) + x][(x^2 + 1) - x] \\
 &\quad (m = x^2 + 1 \text{ મૂકતી}) \\
 &= (x^2 + 1 + x)(x^2 + 1 - x) \\
 &= (x^2 + x + 1)(x^2 - x + 1)
 \end{aligned}$$

મહાવરો 4 [પાન નંબર 55]

અવયવ પાડો :

(1) $a^4 + 6a^2 + 25$

$$\begin{aligned}
 \text{મધ્યમ પદ} &= \pm 2 \times \sqrt{a^4} \times \sqrt{25} \\
 &= \pm 2 \times a^2 \times 5 = \pm 10a^2 \\
 a^4 + 6a^2 + 25 &= a^4 + 10a^2 + 25 - 4a^2 \\
 &= (a^2 + 5)^2 - (2a)^2 \\
 &= (m)^2 - (2a)^2 \quad (a^2 + 5 = m \text{ ધારતી}) \\
 &= (m + 2a)(m - 2a) \\
 &= [(a^2 + 5) + 2a][(a^2 + 5) - 2a] \\
 &\quad (m = a^2 + 5 \text{ મૂકતી}) \\
 &= (a^2 + 5 + 2a)(a^2 + 5 - 2a) \\
 &= (a^2 + 2a + 5)(a^2 - 2a + 5)
 \end{aligned}$$

(2) $m^4 - 7m^2 + 9$

$$\begin{aligned}
 \text{મધ્યમ પદ} &= \pm 2 \times \sqrt{m^4} \times \sqrt{9} \\
 &= \pm 2 \times m^2 \times 3 = \pm 6m^2 \\
 m^4 - 7m^2 + 9 &= m^4 - 6m^2 + 9 - m^2 \\
 &= (m^4 - 6m^2 + 9) - (m^2) \\
 &= (m^2 - 3)^2 - (m)^2 \\
 &= (x)^2 - (m)^2 \quad (m^2 - 3 = x \text{ ધારતી}) \\
 &= (x + m)(x - m) \\
 &= [(m^2 - 3) + m][(m^2 - 3) - m] \\
 &\quad (x = m^2 - 3 \text{ મૂકતી}) \\
 &= (m^2 - 3 + m)(m^2 - 3 - m) \\
 &= (m^2 + m - 3)(m^2 - m - 3)
 \end{aligned}$$

(3) $4x^4 - 21x^2 + 25$

$$\begin{aligned}
 \text{મધ્યમ પદ} &= \pm 2 \times \sqrt{4x^4} \times \sqrt{25} \\
 &= \pm 2 \times 2x^2 \times 5 \\
 &= \pm 20x^2
 \end{aligned}$$

$$\begin{aligned}
4x^4 - 21x^2 + 25 &= 4x^4 - 20x^2 + 25 - x^2 \\
&= (4x^4 - 20x^2 + 25) - (x^2) \\
&= (2x^2 - 5)^2 - (x)^2 \\
&= (m)^2 - (x)^2 \quad (2x^2 - 5 = m \text{ ધારતા}) \\
&= (m + x)(m - x) \\
&= [(2x^2 - 5) + x][(2x^2 - 5) - x] \\
&\quad (m = 2x^2 - 5 \text{ મૂકતા}) \\
&= (2x^2 - 5 + x)(2x^2 - 5 - x) \\
&= (2x^2 + x - 5)(2x^2 - x - 5)
\end{aligned}$$

$$(4) a^4 - 8a^2b^2 + 4b^4$$

$$\begin{aligned}
\text{મધ્યમ પદ} &= \pm 2 \times \sqrt{a^4} \times \sqrt{4b^4} \\
&= \pm 2 \times a^2 \times 2b^2 = \pm 4a^2b^2 \\
a^4 - 8a^2b^2 + 4b^4 &= a^4 - 4a^2b^2 + 4b^4 - 4a^2b^2 \\
&= (a^4 - 4a^2b^2 + 4b^4) - (4a^2b^2) \\
&= (a^2 - 2b^2)^2 - (2ab)^2 \\
&= (m)^2 - (2ab)^2 \quad (a^2 - 2b^2 = m \text{ ધારતા}) \\
&= (m + 2ab)(m - 2ab) \\
&= [(a^2 - 2b^2) + 2ab][(a^2 - 2b^2) - 2ab] \\
&\quad (m = a^2 - 2b^2 \text{ મૂકતા}) \\
&= (a^2 - 2b^2 + 2ab)(a^2 - 2b^2 - 2ab) \\
&= (a^2 + 2ab - 2b^2)(a^2 - 2ab - 2b^2)
\end{aligned}$$

$$(5) y^4 + 3y^2 + 4$$

$$\begin{aligned}
\text{મધ્યમ પદ} &= \pm 2 \times \sqrt{y^4} \times \sqrt{4} \\
&= \pm 2 \times y^2 \times 2 = \pm 4y^2 \\
y^4 + 3y^2 + 4 &= y^4 + 4y^2 + 4 - y^2 \\
&= (y^4 + 4y^2 + 4) - (y^2) \\
&= (y^2 + 2)^2 - (y)^2 \\
&= (m)^2 - (y)^2 \quad (y^2 + 2 = m \text{ ધારતા}) \\
&= (m + y)(m - y) \\
&= [(y^2 + 2) + y][(y^2 + 2) - y] \\
&\quad (m = y^2 + 2 \text{ મૂકતા}) \\
&= (y^2 + 2 + y)(y^2 + 2 - y) \\
&= (y^2 + y + 2)(y^2 - y + 2)
\end{aligned}$$

$$(6) m^4 + 12m^2 + 64$$

$$\begin{aligned}
\text{મધ્યમ પદ} &= \pm 2 \times \sqrt{m^4} \times \sqrt{64} \\
&= \pm 2 \times m^2 \times 8 = \pm 16m^2
\end{aligned}$$

$$\begin{aligned}
m^4 + 12m^2 + 64 &= m^4 + 16m^2 + 64 - 4m^2 \\
&= (m^4 + 16m^2 + 64) - (4m^2) \\
&= (m^2 + 8)^2 - (2m)^2 \\
&= (x)^2 - (2m)^2 \quad (m^2 + 8 = x \text{ ધારતા}) \\
&= (x + 2m)(x - 2m) \\
&= [(m^2 + 8) + 2m][(m^2 + 8) - 2m] \\
&\quad (x = m^2 + 8 \text{ મૂકતા}) \\
&= (m^2 + 8 + 2m)(m^2 + 8 - 2m) \\
&= (m^2 + 2m + 8)(m^2 - 2m + 8)
\end{aligned}$$

નવું શીખીએ ... યાદ રાખીએ

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$\text{તે પરથી } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$\text{તે પરથી } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

મહાવરો 5 [પાન નંબર 56-57]

1. અવયવ પાડો :

$$(1) x^3 + 27$$

$$= (x)^3 + (3)^3$$

$$= (x + 3)[(x)^2 - (x)(3) + (3)^2]$$

$$= (x + 3)(x^2 - 3x + 9)$$

$$(2) a^3 + 125b^3$$

$$= (a)^3 + (5b)^3$$

$$= (a + 5b)[(a)^2 - (a)(5b) + (5b)^2]$$

$$= (a + 5b)(a^2 - 5ab + 25b^2)$$

* પ્રથમ પદ અને અંતિમ પદના સહગુણકોનો ગુણાકાર ($a \times c$) ધન હોય, તો ($a \times c$)ના બે અવયવો એવા મેળવવા કે, જેથી બંને અવયવોનો સરવાળો મધ્યમ પદના સહગુણક જેટલો થાય.

* જો મધ્યમ પદ ધન હોય, તો બંને અવયવ ધન આવે અને મધ્યમ પદ ઋણ હોય, તો બંને અવયવ ઋણ આવે.

* પ્રથમ પદ અને અંતિમ પદના સહગુણકોનો ગુણાકાર ($a \times c$) ઋણ હોય, તો ($a \times c$)ના બે અવયવો એવા મેળવવા કે, જેથી બંને અવયવોની બાદબાકી મધ્યમ પદના સહગુણક જેટલી થાય.

* જો મધ્યમ પદ ધન હોય, તો મોટો અવયવ ધન અને નાનો અવયવ ઋણ લેવાય અને જો મધ્યમ પદ ઋણ હોય, તો મોટો અવયવ ઋણ અને નાનો અવયવ ધન લેવાય.

મહાવરો 6 [પાન નંબર 58-59]

અવયવ પાડો :

$$(1) x^2 + 5x + 6$$

$$= x^2 + 3x + 2x + 6$$

$$= x(x + 3) + 2(x + 3)$$

$$= (x + 3)(x + 2)$$

$$(3) x^2 - 11x + 24$$

$$= x^2 - 3x - 8x + 24$$

$$= x(x - 3) - 8(x - 3)$$

$$= (x - 3)(x - 8)$$

$$(5) x^2 + 6x - 27$$

$$= x^2 + 9x - 3x - 27$$

$$= x(x + 9) - 3(x + 9)$$

$$= (x + 9)(x - 3)$$

$$(7) m^2 - 2m - 8$$

$$= m^2 - 4m + 2m - 8$$

$$= m(m - 4) + 2(m - 4)$$

$$= (m - 4)(m + 2)$$

$$(2) x^2 + 15x + 50$$

$$= x^2 + 10x + 5x + 50$$

$$= x(x + 10) + 5(x + 10)$$

$$= (x + 10)(x + 5)$$

$$(4) x^2 - 7x + 12$$

$$= x^2 - 4x - 3x + 12$$

$$= x(x - 4) - 3(x - 4)$$

$$= (x - 4)(x - 3)$$

$$(6) a^2 + 4a - 21$$

$$= a^2 + 7a - 3a - 21$$

$$= a(a + 7) - 3(a + 7)$$

$$= (a + 7)(a - 3)$$

$$(8) n^2 - 4n - 45$$

$$= n^2 - 9n + 5n - 45$$

$$= n(n - 9) + 5(n - 9)$$

$$= (n - 9)(n + 5)$$

$$(9) 4x^2 + 12x + 5$$

$$= 4x^2 + 10x + 2x + 5$$

$$= 2x(2x + 5) + 1(2x + 5)$$

$$= (2x + 5)(2x + 1)$$

$$(10) 9y^4 - 13y^2 + 4$$

$$= 9y^4 - 9y^2 - 4y^2 + 4$$

$$= 9y^2(y^2 - 1) - 4(y^2 - 1)$$

$$= (y^2 - 1)(9y^2 - 4)$$

$$= (y + 1)(y - 1)(3y + 2)(3y - 2)$$

$$(11) 2a^2 - 19a - 21$$

$$= 2a^2 + 2a - 21a - 21$$

$$= 2a(a + 1) - 21(a + 1)$$

$$= (a + 1)(2a - 21)$$

$$(12) 6x^2 - 7x - 3$$

$$= 6x^2 - 9x + 2x - 3$$

$$= 3x(2x - 3) + 1(2x - 3)$$

$$= (2x - 3)(3x + 1)$$

સ્વાધ્યાય [પાન નંબર 59]

1. પાલી જગ્યા પૂરો :

$$(1) 36 - x^2 = (6 + x)(\underline{6 - x})$$

$$\text{રીત : } 36 - x^2 = (6)^2 - (x)^2$$

$$= (6 + x)(6 - x)$$

$$(2) a^2 - b^2c^2 = (\underline{a - bc})(a + bc)$$

$$\text{રીત : } a^2 - b^2c^2 = (a)^2 - (bc)^2$$

$$= (a + bc)(a - bc)$$

$$(3) x^3 - 49x = \underline{x}(x + 7)(x - 7)$$

$$\text{રીત : } x^3 - 49x = x(x^2 - 49)$$

$$= x(x + 7)(x - 7)$$

$$(4) 4x^2 - 25 = (2x + 5)(\underline{2x - 5})$$

$$\text{રીત : } 4x^2 - 25 = (2x)^2 - (5)^2$$

$$= (2x + 5)(2x - 5)$$

$$(5) x^2 + 5x + 6 = (x + 3)(\underline{x + 2})$$

$$\text{રીત : } x^2 + 5x + 6 = x^2 + 3x + 2x + 6$$

$$= x(x + 3) + 2(x + 3)$$

$$= (x + 3)(x + 2)$$

$$(6) x^2 - x - 12 = (x + 3)(x - 4)$$

$$\begin{aligned} \text{રીત : } x^2 - x - 12 &= x^2 + 3x - 4x - 12 \\ &= x(x + 3) - 4(x + 3) \\ &= (x + 3)(x - 4) \end{aligned}$$

$$(7) a^3 - 1 = (a - 1)(a^2 + a + 1)$$

$$\begin{aligned} \text{રીત : } a^3 - 1 &= (a)^3 - (1)^3 \\ &= (a - 1)[(a)^2 + (a)(1) + (1)^2] \\ &= (a - 1)(a^2 + a + 1) \end{aligned}$$

$$(8) m^3 + 125 = (m + 5)(m^2 - 5m + 25)$$

$$\begin{aligned} \text{રીત : } m^3 + 125 &= (m)^3 + (5)^3 \\ &= (m + 5)[(m)^2 - (m)(5) + (5)^2] \\ &= (m + 5)(m^2 - 5m + 25) \end{aligned}$$

2. અવયવ પાડો :

$$(1) 16a^2b^2 - 36$$

$$\begin{aligned} &= 4(4a^2b^2 - 9) \\ &= 4[(2ab)^2 - (3)^2] \\ &= 4(2ab - 3)(2ab + 3) \end{aligned}$$

$$\text{અથવા } (1) 16a^2b^2 - 36$$

$$\begin{aligned} &= (4ab)^2 - (6)^2 \\ &= (4ab - 6)(4ab + 6) \\ &= [2(2ab - 3)][2(2ab + 3)] \\ &= 4(2ab - 3)(2ab + 3) \end{aligned}$$

$$(2) 625 - 64x^2$$

$$\begin{aligned} &= (25)^2 - (8x)^2 \\ &= (25 + 8x)(25 - 8x) \end{aligned}$$

$$(3) 4x^5 - 64x$$

$$\begin{aligned} &= 4x(x^4 - 16) \\ &= 4x[(x^2)^2 - (4)^2] \\ &= 4x(x^2 + 4)(x^2 - 4) \\ &= 4x(x^2 + 4)(x + 2)(x - 2) \end{aligned}$$

$$(4) (4a - 5b)^2 - 16c^2$$

$$\begin{aligned} &= (4a - 5b)^2 - (4c)^2 \\ &= (4a - 5b + 4c)(4a - 5b - 4c) \end{aligned}$$

$$(5) 25 - (ab - 3x)^2$$

$$\begin{aligned} &= (5)^2 - (ab - 3x)^2 \\ &= (5)^2 - (m)^2 \quad (ab - 3x = m \text{ ધારતી}) \\ &= (5 + m)(5 - m) \\ &= [5 + (ab - 3x)][5 - (ab - 3x)] \quad (m = ab - 3x \text{ મૂકતી}) \\ &= (5 + ab - 3x)(5 - ab + 3x) \end{aligned}$$

$$(6) (x + 8)^2 - (2x - 3)^2$$

$$\begin{aligned} &= (m)^2 - (n)^2 \quad (x + 8 = m \text{ અને } 2x - 3 = n \text{ ધારતી}) \\ &= (m + n)(m - n) \\ &= [(x + 8) + (2x - 3)][(x + 8) - (2x - 3)] \\ & \quad (m = x + 8 \text{ અને } n = 2x - 3 \text{ મૂકતી}) \\ &= (x + 8 + 2x - 3)(x + 8 - 2x + 3) \\ &= (3x + 5)(-x + 11) \end{aligned}$$

$$(7) 121x^2 - 22x + 1 - 9a^2 - 24ab - 16b^2$$

$$\begin{aligned} &= (121x^2 - 22x + 1) - (9a^2 + 24ab + 16b^2) \\ &= (11x - 1)^2 - (3a + 4b)^2 \\ &= (m)^2 - (n)^2 \quad (11x - 1 = m \text{ અને } 3a + 4b = n \text{ ધારતી}) \\ &= (m + n)(m - n) \\ &= [(11x - 1) + (3a + 4b)][(11x - 1) - (3a + 4b)] \\ & \quad (m = 11x - 1 \text{ અને } n = 3a + 4b \text{ મૂકતી}) \\ &= (11x - 1 + 3a + 4b)(11x - 1 - 3a - 4b) \\ &= (11x + 3a + 4b - 1)(11x - 3a - 4b - 1) \end{aligned}$$

$$(8) x^4 + 4y^4$$

$$\begin{aligned} \text{મધ્યમ પદ} &= \pm 2 \times \sqrt{x^4} \times \sqrt{4y^4} \\ &= \pm 2 \times x^2 \times 2y^2 \\ &= \pm 4x^2y^2 \end{aligned}$$

$$\begin{aligned} &= x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2 \\ &= (x^2 + 2y^2)^2 - (2xy)^2 \\ &= (x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy) \\ &= (x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2) \end{aligned}$$

$$\therefore x = 6 \times 4$$

$$\therefore x = 24$$

\therefore ધારેલી સંખ્યા 24 છે.

(5) ધારેલી સંખ્યાના પાંચમા ભાગમાંથી 1 બાદ કરતાં 3 મળે છે.

$$\frac{x}{5} - 1 = 3$$

$$\therefore \frac{x}{5} = 3 + 1$$

$$\therefore \frac{x}{5} = 4$$

$$\therefore x = 4 \times 5$$

$$\therefore x = 20$$

\therefore ધારેલી સંખ્યા 20 છે.

$ax + b = cx + d$ પ્રકારનાં સમીકરણોનો ઉકેલ

મહાવરો 1 [પાન નંબર 66-67]

નીચેનાં સમીકરણ ઉકેલો :

$$(1) 3x + 8 = -5x + 4$$

$$\therefore 3x + 8 + 5x = 4$$

$$\therefore 8x + 8 = 4$$

$$\therefore 8x = 4 - 8$$

$$\therefore 8x = -4$$

$$\therefore x = \frac{-4}{8}$$

$$\therefore x = \left(-\frac{1}{2}\right)$$

\therefore આપેલ સમીકરણનો ઉકેલ $\left(-\frac{1}{2}\right)$ છે.

$$(2) \frac{x}{2} - 2 = \frac{x}{3} + 1$$

$$\therefore \frac{x}{2} - \frac{x}{3} - 2 = 1$$

$$\therefore \frac{x}{2} - \frac{x}{3} = 1 + 2$$

$$\therefore \frac{x}{2} - \frac{x}{3} = 3$$

$$\therefore 6\left(\frac{x}{2}\right) - 6\left(\frac{x}{3}\right) = 6(3) \quad (\text{બંને બાજુ 6 વડે ગુણતાં})$$

$$\therefore 3x - 2x = 18$$

$$\therefore x = 18$$

\therefore આપેલ સમીકરણનો ઉકેલ 18 છે.

$$(3) 5x + \frac{7}{2} = \frac{3x}{2} - 14$$

$$\therefore 5x - \frac{3x}{2} = -14 - \frac{7}{2}$$

$$\therefore 2\left(5x - \frac{3x}{2}\right) = 2\left(-14 - \frac{7}{2}\right) \quad (\text{બંને બાજુ 2 વડે ગુણતાં})$$

$$\therefore 10x - 3x = -28 - 7$$

$$\therefore 7x = -35$$

$$\therefore x = \frac{-35}{7} \quad \therefore x = (-5)$$

\therefore આપેલ સમીકરણનો ઉકેલ (-5) છે.

$$(4) x = \frac{4}{5}(x + 10)$$

$$\therefore 5x = 4(x + 10) \quad (\text{બંને બાજુ 5 વડે ગુણતાં})$$

$$\therefore 5x = 4x + 40$$

$$\therefore 5x - 4x = 40$$

$$\therefore x = 40$$

\therefore આપેલ સમીકરણનો ઉકેલ 40 છે.

$$(5) 2y + \frac{5}{3} = \frac{26}{3} - y$$

$$2y + y = \frac{26}{3} - \frac{5}{3}$$

$$\therefore 3y = \frac{26-5}{3}$$

$$\therefore 3y = \frac{21}{3}$$

$$\therefore 3y = 7$$

$$\therefore y = \frac{7}{3}$$

$$\therefore y = 2\frac{1}{3}$$

\therefore આપેલ સમીકરણનો ઉકેલ $2\frac{1}{3}$ છે.

$$(6) \frac{2x}{3} + 1 = \frac{7x}{15} + 3$$

$$\therefore \frac{2x}{3} - \frac{7x}{15} + 1 = 3$$

$$\therefore \frac{2x}{3} - \frac{7x}{15} = 3 - 1$$

$$\therefore \frac{2x}{3} - \frac{7x}{15} = 2$$

$$\therefore 15 \left(\frac{2x}{3} \right) - 15 \left(\frac{7x}{15} \right) = 2 \times 15 \quad (\text{બંને બાજુ 15 વડે ગુણતાં})$$

$$\therefore 5(2x) - 7x = 30$$

$$\therefore 10x - 7x = 30$$

$$\therefore 3x = 30$$

$$\therefore x = \frac{30}{3}$$

$$\therefore x = 10$$

\therefore આપેલ સમીકરણનો ઉકેલ **10** છે.

$$(7) 3m = 5m - \frac{8}{5}$$

$$\therefore 3m - 5m = -\frac{8}{5}$$

$$\therefore -2m = -\frac{8}{5}$$

$$\therefore 2m = \frac{8}{5}$$

$$\therefore m = \frac{8}{5} \times \frac{1}{2}$$

$$\therefore m = \frac{4}{5}$$

\therefore આપેલ સમીકરણનો ઉકેલ $\frac{4}{5}$ છે.

$$(8) 8x + 4 = 3(x - 1) + 7$$

$$\therefore 8x + 4 = 3x - 3 + 7$$

$$\therefore 8x + 4 = 3x + 4$$

$$\therefore 8x - 3x + 4 = 4$$

$$\therefore 5x + 4 = 4$$

$$\therefore 5x = 4 - 4$$

$$\therefore 5x = 0$$

$$\therefore x = \frac{0}{5}$$

$$\therefore x = 0$$

\therefore આપેલ સમીકરણનો ઉકેલ **0** છે.

$$(9) \frac{1}{9}(2x - 1) - \frac{1}{4}(2x + 1) = \frac{1}{12}(2 - x)$$

$$\therefore 4(2x - 1) - 3(2x + 1) = 1(2 - x) \quad (\text{બંને બાજુ 12 વડે ગુણતાં})$$

$$\therefore 8x - 4 - 6x - 3 = 2 - x$$

$$\therefore 2x - 7 = 2 - x$$

$$\therefore 2x + x - 7 = 2$$

$$\therefore 2x + x = 2 + 7$$

$$\therefore 3x = 9$$

$$\therefore x = \frac{9}{3}$$

$$\therefore x = 3$$

\therefore આપેલ સમીકરણનો ઉકેલ **3** છે.

$$(10) 9x - 4 - 3(x - 4) = 4(x - 1)$$

$$\therefore 9x - 4 - 3x + 12 = 4x - 4$$

$$\therefore 6x + 8 = 4x - 4$$

$$\therefore 6x - 4x + 8 = -4$$

$$\therefore 6x - 4x = -8 - 4$$

$$\therefore 2x = -12$$

$$\therefore x = \frac{-12}{2}$$

$$\therefore x = (-6)$$

\therefore આપેલ સમીકરણનો ઉકેલ **(-6)** છે.

$$(11) \frac{x}{2} + \frac{x+2}{3} + \frac{x+7}{4} = x$$

$$\therefore 12 \left(\frac{x}{2} \right) + 12 \left(\frac{x+2}{3} \right) + 12 \left(\frac{x+7}{4} \right) = 12 \times x$$

(બંને બાજુ 12 વડે ગુણતાં)

$$\therefore 6x + 4(x+2) + 3(x+7) = 12x$$

$$(3) \frac{3a-4}{2-6a} = \frac{-2}{5}$$

$$\therefore 5(3a-4) = -2(2-6a) \quad (\text{ચોકડી ગુણાકાર કરતાં})$$

$$\therefore 15a - 20 = -4 + 12a$$

$$\therefore 15a - 12a = -4 + 20$$

$$\therefore 3a = 16$$

$$\therefore a = \frac{16}{3}$$

$$\therefore a = 5\frac{1}{3}$$

આપેલ સમીકરણનો ઉકેલ $5\frac{1}{3}$ છે.

$$(4) \frac{m}{m+15} = \frac{4}{9}$$

$$\therefore 9m = 4(m+15) \quad (\text{ચોકડી ગુણાકાર કરતાં})$$

$$\therefore 9m = 4m + 60$$

$$\therefore 9m - 4m = 60$$

$$\therefore 5m = 60$$

$$\therefore m = \frac{60}{5}$$

$$\therefore m = 12$$

આપેલ સમીકરણનો ઉકેલ 12 છે.

$$(5) \frac{7n+4}{n+2} = \frac{-4}{3}$$

$$\therefore 3(7n+4) = -4(n+2) \quad (\text{ચોકડી ગુણાકાર કરતાં})$$

$$\therefore 21n + 12 = -4n - 8$$

$$\therefore 21n + 4n = -8 - 12$$

$$\therefore 25n = -20$$

$$\therefore n = \frac{-20}{25}$$

$$\therefore n = \left(\frac{-4}{5}\right)$$

આપેલ સમીકરણનો ઉકેલ $\left(\frac{-4}{5}\right)$ છે.

$$(6) \frac{3}{2} \left(\frac{4x+1}{2x-5} \right) = \frac{3}{2}$$

$$\therefore \frac{4x+1}{2x-5} = 1$$

(બંને બાજુ $\frac{2}{3}$ વડે ગુણતાં)

$$\therefore 4x + 1 = 2x - 5$$

(ચોકડી ગુણાકાર કરતાં)

$$\therefore 4x - 2x = -5 - 1$$

$$\therefore 2x = -6$$

$$\therefore x = \frac{-6}{2}$$

$$\therefore x = (-3)$$

આપેલ સમીકરણનો ઉકેલ (-3) છે.

$$(7) \frac{4x-1}{2x+1} = \frac{6x-5}{3x+2}$$

$$\therefore (4x-1)(3x+2) = (6x-5)(2x+1) \quad (\text{ચોકડી ગુણાકાર કરતાં})$$

$$\therefore 12x^2 + 8x - 3x - 2 = 12x^2 + 6x - 10x - 5$$

$$\therefore 12x^2 + 5x - 2 = 12x^2 - 4x - 5$$

$$\therefore 12x^2 - 12x^2 + 5x + 4x = -5 + 2$$

$$\therefore 9x = -3$$

$$\therefore x = \frac{-3}{9}$$

$$\therefore x = \left(\frac{-1}{3}\right)$$

આપેલ સમીકરણનો ઉકેલ $\left(\frac{-1}{3}\right)$ છે.

$$(8) \frac{4x-1}{4x+1} = \frac{2x+1}{2x-1}$$

$$\therefore (4x-1)(2x-1) = (2x+1)(4x+1) \quad (\text{ચોકડી ગુણાકાર કરતાં})$$

$$\therefore 8x^2 - 4x - 2x + 1 = 8x^2 + 2x + 4x + 1$$

$$\therefore 8x^2 - 6x + 1 = 8x^2 + 6x + 1$$

$$\therefore 8x^2 - 8x^2 - 6x - 6x = 1 - 1$$

$$\therefore -12x = 0$$

$$\therefore x = \frac{0}{-12}$$

$$\therefore x = 0$$

આપેલ સમીકરણનો ઉકેલ 0 છે.